Approximation by integral operators as tool of signal analysis and image processing

Gianluca Vinti, *University of Perugia, Italy*

In the last century, Whittaker, Kotelnikov and Shannon stated the celebrated WKS-sampling theorem, which can be formulate as follows ([3]):

let $f \in L^2(\mathbb{R})$ be a function with the support of its Fourier transform $\hat{f}$ contained in an interval $[-\pi w, \pi w]$, for $w > 0$, (i.e. f is a *band-limited* function); then f can be completely reconstructed on the whole real time-axis from its samples values by means of the interpolation series:

$$f(t) = \sum_{k=-\infty}^{+\infty} f(\frac{k}{w}) sinc[\pi(wt - k)], \; t \in \mathbb{R}$$

where $sinc(t) = \frac{\sin t}{t}, t \neq 0$ and $sinc(0) = 1$.
Although this theorem had a strong impact in communication theory as in image processing, the above interpolation formula has some disadvantages in terms of its applications, as it will be showed along this talk; moreover the band-limitation to $[-\pi w, \pi w]$ is a rather restrictive condition. Several contributions have been given in order to weaken the band-limitation, but the most important contribution, based on an approximation theory's approach, has been given by P.L.Butzer and his school at Aachen ([2]) considering a family of discrete operators, called "generalized sampling series" of the form

$$(S_w^{\varphi} f)(t) := \sum_{k=-\infty}^{\infty} f(\frac{k}{w}) \; \varphi(wt - k), \quad t \in \mathbb{R}, \; k \in Z, \; w > 0$$

where φ is a continuous function with compact support on $\mathbb{R}$.
For the above operators, pointwise and uniform convergence results for continuous signals have been obtained together with the study of the rate of approximation under suitable singularity assumptions on the family of kernels involved. The interest of the Butzer's approach relies not only in its mathematical setting, but also in its concrete applications. Moreover some recent results in L^p-setting will be discussed.

In this talk I will show how the Butzer's approach can be viewed as a particular case of a more general theory based on functions defined on locally compact topological groups and dealing with Haar integrals, which seems to be the new tendency in signal processing. This new approach allows us to treat several classes of integral operators that include, in particular, the generalized sampling series introduced by Butzer. An advantage of this approach, is the formulation of the theory in general functional spaces which allows to cover the cases of L^p-spaces, Orlicz type spaces and even modular spaces, so obtaining convergence results and rate of approximation in this new frame. Due to this setting, it is possible to built a linear and a nonlinear sampling theory which goes beyond the applications to signals which are not necessarily of finite energy or band-limited, as in Butzer's approach, to include signals that are discontinuous. The reconstruction of discontinuous signals plays an important rule in image enhancement where the jump of grey levels which occur in the edge of the image (discontinuities) can be described by BV functions.

References

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Prof. Gianluca Vinti
Dipartimento di Matematica e Informatica
Università degli Studi di Perugia
Via Vanvitelli,1
06123 PERUGIA
ITALY
Phone:(075) 5855025-5853822
Fax:(075) 5855024-5853822
e-mail: mategian@unipg.it