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Topological and geometric structure of Cesaro-Orlicz spaces.

The space of all real sequences $x = (x(i))_{i=1}^{\infty}$ is denoted by l^0. A map $\varphi : \mathbb{R} \to [0, \infty]$ is said to be an Orlicz function if φ is even, convex, left continuous on $\mathbb{R}_+$, continuous at zero, $\varphi(0) = 0$ and $\varphi(u) \to \infty$ as $u \to \infty$. Given any Orlicz function φ, we define on l^0 the following modular

$$\rho_\varphi(x) = \sum_{n=1}^{\infty} \varphi\left(\frac{1}{n}\sum_{i=1}^{n}|x(i)|\right).$$

The space

$$\mathrm{ces}_\varphi = \left\{x \in l^0 : \exists_{k>0}\ \rho_\varphi(kx) < \infty\right\}$$

where φ is an Orlicz function is called the Cesaro-Orlicz space. We equip this space with the Luxemburg norm

$$\|x\|_\varphi = \inf\left\{\lambda > 0 : \rho_\varphi\left(\frac{x}{\lambda}\right) \leqslant 1\right\}.$$

Necessary and sufficient conditions for nontriviality of ces_φ are presented in the following

Theorem 1 *The following conditions are equivalent:*

1. $\mathrm{ces}_\varphi \neq \{0\}$,
2. $\exists_{n_1} \sum_{n=n_1}^{\infty} \phi\left(\frac{1}{n}\right) < \infty$,
3. $\forall_k > 0 \quad \exists_{n_k} \sum_{n=n_k}^{\infty} \phi\left(\frac{k}{n}\right) < \infty$.

We define the following subspaces of the Cesaro-Orlicz space:

$$A_\varphi = \left\{x \in \mathrm{ces}_\varphi : \forall_{k>0} \quad \exists_{n_k\in\mathbb{N}} \sum_{n=n_k}^{\infty} \varphi\left(\frac{k}{n}\sum_{i=1}^{n}|x(i)|\right) < \infty\right\}$$

and

$$\mathrm{ces}_\varphi^n = \{x \in \mathrm{ces}_\varphi : \forall_{k>n} \quad x(k) = 0\}.$$

The criteria for rotundity of these subspaces are given in the followings:

Theorem 2 *The following conditions are equivalent:*

1. ces_φ^2 *is rotund,*
2. $\varphi > 0$ *and* $\sum_{i=1}^{\infty} \varphi\left(\frac{b_\varphi}{i}\right) \geqslant 1$, *where* $b_\varphi := \sup\{u \geqslant 0 : \varphi(u) < \infty\}$.

Theorem 3 *The following conditions are equivalent:*

1. A_φ *is rotund,*
2. ces_φ^n *is rotund for* $n \geqslant 3$,
3. φ *is strictly convex on the interval* $[0, \alpha]$, *where* $\alpha > 0$ *is a solution of an equation* $2\varphi(x) + \sum_{n=3}^{\infty} \varphi\left(\frac{2}{i}x\right) = 1$.