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A converse to Fubini's theorem

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Abstract

Let Ω be a set, let $\mathcal{M}$ be a sigma algebra of subsets of Ω and let μ be a nonnegative additive set function defined in $\mathcal{M}$. Further, let X be a normed space of measurable real functions defined in a set Ω, suppose that X contains characteristic functions of sets from $\mathcal{M}$ and let $M : X \longrightarrow \mathbb{R}$ be a continuous and reflexive functional. Assume that M is strictly increasing i. e. $Mf > Mg$ whenever $f \geqslant g$ and $\mu(\{f > g\}) > 0$. We prove that there exist a positive additive set function P and a strictly increasing continuous function $\varphi : \mathbb{R} \longrightarrow \mathbb{R}$ such that $(P(A) > 0 \iff \mu(A) > 0)$ and $Mf = \varphi^{-1}\left(\int_\Omega \varphi \circ f dP\right)$ if and only if for every function $x : \Omega \times \Omega \longrightarrow \mathbb{R}$ such that $x(s, \cdot)$ and $x(\cdot, t)$ belong to X, as well as $M_{[t]}x :=$ "$s \to Mx(s, \cdot)$"and $M_{[s]}x :=$ "$t \to Mx(\cdot, t)$", we have

$$M\left(M_{[s]}x\right) = M\left(M_{[t]}x\right).$$