ON CHAOTIC MAPS IN BI-INFINITE SYMBOL SPACE

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Dynamical systems originally arose in the study of systems of differential equations used to model physical phenomena. The technique of characterizing the orbit structure of a dynamical system via infinite sequences of "symbols" (in our case "0" and "1") is known as *symbolic dynamics*. Well known is shift map, which is a chaotic map in symbolic space. But there exist other chaotic maps in symbol space as well. We consider some continious mappings and show that they are chaotic.

Definition 1 The set of all bi-infinite sequences of 0s and 1s is called a sequence space of 0 and 1 or the *symbolic space* of 0 and 1, i.e.,

$$\Sigma = \{\ ...s_{-2}s_{-1}\ .s_0s_1s_2...\ |s_i = 0 \text{ or } s_i = 1 \text{ for every } i \in \mathbf{Z}\}.$$

We use following definition of R.Devaney ([1]). Let (X, ρ) be metric space.

Definition 2 ([1]): The function $f : X \to X$ is *chaotic* if
a) the periodic points of f are dense in X,
b) f is topologically transitive,
c) f exhibits sensitive dependence on initial conditions.

Definition 3 The map $\alpha : \Sigma \to \Sigma$ is called an *α-map* if

$$\forall\ s = ...s_{-2}s_{-1}\ .s_0s_1s_2... \in \Sigma :\ \alpha(s) = ...s_{-n}...s_{-3}s_{-2}\ .s_1s_2...s_n....$$

Definition 4 The map $\alpha_{mk} : \Sigma \to \Sigma$ $(m, k \in \mathbf{N})$ is called a *α_{mk}-map* if

$$\forall\ s = ...s_{-2}s_{-1}\ .s_0s_1s_2... \in \Sigma : \quad \alpha_{mk}(s) = ...s_{-(m+2)}s_{-(m+1)}\ .s_ks_{k+1}....$$

Definition 5 The map $\beta : \Sigma \to \Sigma$ is called a *β-map* if

$$\forall\ s = ...s_{-2}s_{-1}\ .s_0s_1s_2... \in \Sigma : \quad \beta(s) = ...s_{-n}...s_{-5}s_{-4}s_{-2}.s_1s_3s_4...s_n....$$

Theorem 1 The α-map, α_{mk}-map and β-map are chaotic in symbol space Σ.

Definition 6 The map $\gamma : \Sigma \to \Sigma$ is called a *γ-map* if

$$\forall\ s = ...s_{-2}s_{-1}\ .s_0s_1s_2... \in \Sigma : \quad \gamma(s) = ...s_{-n}...s_{-3}s_{-2}s_{-1}\ .s_1s_2s_3...s_n....$$

Theorem 2 γ-map and $\overline{\gamma}$-map is not chaotic in symbol space Σ.

References

[1] R.Devaney, *An introduction to chaotic dynamical systems.* Benjamin\Cummings: Menlo Park, CA, 1986.