Characterization of tame pairs (X, Y) for power series spaces

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Abstract:

Let X, Y be Fréchet spaces with the sequences $(||\cdot||_n)_n, (|\cdot|_n)_n$ of seminorms defining their topologies. It is well known that for every operator $T \in L(X, Y)$ there exists a nondecreasing function $\sigma : \mathbb{N} \to \mathbb{N}$ such that

$$|Tx|_k \leq C_k ||x||_{\sigma(k)}. \tag{1}$$

We ask whether there exists a 'universal' function $\psi : \mathbb{N} \to \mathbb{N}$ suitable in (1) for every operator T at least for big k. In such a case (X, Y) is called a tame pair and if $X = Y$ then X is a tame space. These spaces appeared in a paper of Vogt and Dubinsky [1], where it is proved that in a tame power series space of infinite type every complemented subspace has a basis. This solves the problem of Pełczyński in such spaces and gives a motivation to investigate tame spaces.

We characterize tame pairs (X, Y) where at least one of the spaces is a power series space (e.g. the space of holomorphic functions $H(\mathbb{D}^n), H(\mathbb{C}^n)$ or the Schwartz space $\mathcal{S}$ of rapidly decreasing functions). For instance, we show that if X or $Y = H(\mathbb{C}^n)$ then the pair (X, Y) is tame if and only if every operator $T \in L(X, Y)$ maps some 0-neighbourhood into a bounded set.

References

[1] E. Dubinsky, D. Vogt, *Complemented subspaces in tame power series spaces*, Studia Math. 93 (1989), 71–85.