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C–subgradients for locally Lipschitz integral functionals on non-L_p-type spaces of measurable functions

Let (Ω, μ) be a measure space, E be an arbitrary separable Banach space, $E^*_{\omega^*}$ be the dual equipped with the weak* topology, and $g : \Omega \times E \to \mathbb{R}$ be a Carathéodory function which is Lipschitz continuous on each ball of E for almost all $s \in \Omega$. Put $G(x) \stackrel{\text{def}}{=} \int_\Omega g(s, x(s))\, d\mu(s)$. Consider the integral functional G defined on some non-L_p-type Banach space X of measurable functions $x \colon \Omega \to E$. We present several general theorems on sufficient conditions under which any element $\gamma \in X^*$ of Clarke's generalized gradient (multivalued C-subgradient) $\partial G(x)$ has the representation $\gamma(v) = \int_\Omega \langle \zeta(s), v(s) \rangle\, d\mu(s)\ (v \in X)$ via some measurable function $\zeta \colon \Omega \to E^*_{w^*}$ of the associate space X' such that $\zeta(s) \in \partial_u g(s, x(s))$ for almost all $s \in \Omega$. Here, given a fixed $s \in \Omega$, $\partial_u g(s, u_0)$ denotes Clarke's generalized gradient for the function $g(s, \cdot)$ at $u_0 \in E$. What concerning X, we suppose that it is either a so-called "non-solid" Banach M-space (in particular, "non-solid" generalized Orlicz space) or Köthe–Bochner space ("solid" space).