ABSTRACT[On the interpolation theorems related to Banach spaces $CMO^p$, Katsuo MATSUOKA (College of Economics, Nihon University, Japan)]:

C. Fefferman and E. M. Stein proved the interpolation theorem between $L^p(\mathbb{R}^n)$ and John-Nirenberg's $BMO$. Y. Chen and K. Lau, and J. Garcia-Cuerva, defined the spaces $CMO^p$, i.e. the class of functions of central mean oscillation of order $p$, which are similar to $BMO$.

First, we show the interpolation theorem between the non-homogeneous Herz space $B^p = K_p^{-n/p,\infty}$ and $CMO^p$, which is the analogue of the above theorem of C. Fefferman and E. M. Stein, i.e.

**Theorem 1.** *Suppose $1 < p_0 < \infty$, and let $T$ be a linear operator such that*

$$T : B^{p_0} \to B^{p_0}$$

*and*

$$T : L^\infty(\mathbb{R}^n) \to BMO$$

*boundedly. Then, for every $p$ with $p_0 < p < \infty$,*

$$T : B^p \to CMO^p$$

*boundedly.*

Next, we show the analogue of the interpolation theorem for analytic families of operators obtained by C. Fefferman and E. M. Stein, i.e.

**Theorem 2.** *Suppose $1 < p_0 < \infty$, and let $\{T_z\}$, $z \in S = \{z \in \mathbb{C} : 0 \leq \operatorname{Re} z \leq 1\}$, be an admissible family of bounded linear operators from $B^{p_0}$ to $CMO^{p_0}$, i.e. the function $z \mapsto \int_{\mathbb{R}^n} (T_z f)(x) g(x) dx$ is analytic in $S_0 = \{z \in \mathbb{C} : 0 < \operatorname{Re} z < 1\}$ and continuous in $S$ for all $f \in B^{p_0}$ and $g \in HA^{p_0'}$, where $1/p_0 + 1/p_0' = 1$, and there exists a positive constant $a < \pi$ such that*

$$\sup_{z \in S} e^{-a|\operatorname{Im} z|} \log \left| \int_{\mathbb{R}^n} (T_z f)(x) g(x) dx \right| < \infty.$$

*And suppose*

$$\|T_{it} f\|_{CMO^{p_0}} \leq A_0(t) \|f\|_{B^{p_0}}$$

*for all $f \in B^{p_0}$ and*

$$\|T_{1+it} f\|_{BMO} \leq A_1(t) \|f\|_{L^\infty}$$

*for all $f \in L^\infty$, where $-\infty < t < \infty$, $A_j(t)$, $j = 0, 1$, are independent of $f$ and satisfy*

$$\sup_{-\infty < t < \infty} e^{-b|t|} \log A_j(t) < \infty$$

*for some positive constant $b < \pi$. Then, for each $\theta$ with $0 < \theta < 1$, we have*

$$\|T_\theta f\|_{CMO^p} \leq A_\theta \|f\|_{B^p}$$

*for all $f \in B^p$, where $1/p = (1-\theta)/p_0$ and $A_\theta$ depends only on $A_j(t)$, $j = 0, 1$, and $\theta$.*

These theorems are proved using the sharp function and the duality.