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Interpolation between L_p and L_q spaces

Lech Maligranda

Department of Mathematics, Luleå University of Technology
SE-971 87 Luleå, SWEDEN, *e-mail: lech@sm.luth.se*
Website: http://www.sm.luth.se/∼lech/ http://www.ltu.se/inst/mat/staff/1.2051

The four cases are important in these investigations: 1) $p = 1$, $q = \infty$, 2) $1 < p < \infty$, $q = \infty$, 3) $p = 1$, $1 < q < \infty$ and 4) $1 < p < q < \infty$. The first two are descibed in [3, 5]. Our special attention will be taken about the other two cases.

We show that if X is a symmetric space on $[0, 1]$ which is an interpolation space between L_1 and L_∞ and for which we have only one-sided estimate of the Boyd index $\alpha(X) > 1/q$, $1 < q < \infty$, then X is an interpolation space between L_1 and L_q. This gives a positive answer for a question posed by E. M. Semenov (cf. [1, 2]).

We also present strong interpolation spaces between L_p and L_q spaces. The notion of p-convexity and q-concavity of Banach (or quasi-Banach) function spaces will be used (cf. [6, 7]).

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