Existence and one-complemented subspaces

Grzegorz Lewicki
Institute of Mathematics, Jagiellonian University,
30-059 Kraków, Reymonta 4, Poland

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Abstract

Let X be a Banach space and let $Y \subset X$ be a linear, closed subspace of X. Denote by $\mathcal{P}(X, Y)$ the set of all continuous, linear projections from X onto Y. A subspace $Y \subset X$, $Y \neq \{0\}$ is *called one-complemented* if there exists $P_o \in \mathcal{P}(X, Y)$, $\|P_o\| = 1$. Y is called an *existence subspace* if for any $x \in X \setminus Y$ there exists a norm-one projection $P_x : Y \oplus [x] \to Y$. It is clear that if Y is one-complemented then Y is an existence subspace. In [**?**], Prop. 5, the opposite implication has been proved for smooth Banach spaces X. Also in [**?**] an example of an existence subspace which is not one-complemented has been presented.
In my talk I will show a generalization of [**?**], Prop. 5. Next I will apply this result to show that in non-smooth spaces c_o, c, l_1, non-smooth Musielak-Orlicz sequence spaces l_Φ (under some assumptions on Φ) and Lorentz sequence spaces $d(w, 1)$ any existence subspace is one-complemented. These are joint results with A. Kamińska, Han Ju Lee and G. Trombetta (see [**?**] and [**?**]).

References

[1] B. Beuzamy and B. Maurey, *Points minimaux et ensambles optimaux dans les espaces de Banach,* J. Funct. Anal. **24** (1977), 107 - 139.

[2] G. Lewicki and G. Trombetta *Optimal and one-complemented subspaces,* IMUJ Preprint 2005/12.

[3] A. Kamińska, Han Ju Lee and G. Lewicki *Extreme and smooth points in Lorentz and Marcinkiewicz spaces,* IMUJ Preprint 2006/01.

[4] J. Lindenstrauss, *On projections with norm 1 - an example,* Proc. Amer. Math. Soc. **15** (1964) 403 -406.