Pointwise strong and very strong (C,α) approximation of Fourier series

Włodzimierz Łenski & Bogdan Roszak
University of Zielona Góra
Faculty of Mathematics, Informatics and Econometry
65-516 Zielona Góra, ul. Szafrana 4a
W.Lenski@wmie.uz.zgora.pl B.Roszak@wmie.uz.zgora.pl

We present an estimation of the $H_{k_0,k_r}^{q,\alpha}f$ and $H_{\lambda,u}^{\phi,\alpha}f$ means of the Fourier series of a function $f \in L^p$ $(1 < p < \infty)$ or $f \in C$ as approximation versions of the Totik type (see [3, 4]) generalization of the result of G. H. Hardy and J. E. Littlewood (see [1]), where the partial sums $S_k f$ are replaced by the (C,α)-means $\sigma_k^\alpha f$ of its Fourier series, i. e.:

$$H_{k_0,k_r}^{q,\alpha}f(x) := \left\{ \frac{1}{r+1} \sum_{\nu=0}^{r} \left| \sigma_{k_\nu}^\alpha f(x) - f(x) \right|^q \right\}^{1/q}, \qquad (q > 0, \quad \alpha > -1).$$

where $0 < k_0 < k_1 < k_2 < ... < k_r$, and

$$H_{\lambda,u}^{\phi,\alpha}f(x) := \sum_{\nu=0}^{\infty} \lambda_\nu(u)\, \phi\left(\left|\sigma_\nu^\alpha f(x) - f(x)\right|\right), \qquad (\alpha > -1)$$

where (λ_ν) is a sequence of positive functions defined on the set having at least one limit point and $\phi : [0,\infty) \to \mathbf{R}$. .

As a measure of approximation by the above quantities we use the pointwise characteristic

$$w_x f(\delta)_p := \left\{ \frac{1}{\delta} \int_0^\delta |\varphi_x(t)|^p \, dt \right\}^{1/p},$$

$$\text{where} \quad \varphi_x(t) := f(x+t) + f(x-t) - 2f(x) \quad ,$$

constructed on the base of definition of Lebesgue points ($L^p - points$) (cf [2]).

We also give some corollaries on norm approximation.

References

[1] G.H. Hardy, J.E. Littlewood, Sur la série de Fourier d'une function a caré sommable, Comptes Rendus, Vol.28,.(1913), 1307-1309.

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[3] V. Totik, On Generalization of Fejér summation theorem, Coll. Math. Soc. J. Bolyai 35 Functions series, operators, Budapest (Hungary) (1980),1195-1199.

[4] V. Totik, Notes on Fourier series: Strong approximation, J.Approx.Th. 43 (1985), 105-111.