Whitney-Besicovitch Coverings and Applications

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Abstract

We will say that a family of cubes $\{K_i\}_{i\in I}$ in $\mathbb{R}^n$ with sides parallell to the coordinate axes forms a Whitney-Besicovitch covering (WB-covering) if the following three properties hold:

A) $\sum_i \chi_{K_i} \leq M(n)$ (*finite multiplicity*);

B) $\cup_i \frac{1}{2}K_i = \cup_i K_i$ (*strong covering*);

C) if $K_i \cap K_j \neq \emptyset$, then $|K_i \cap K_j| \geq \varepsilon(n)\max(|K_i|, |K_j|)$ (*strong linkedness*),

where $M(n), \varepsilon(n)$ are some positive constants depending only on the dimension n.

In the talk I plan to discuss

a) how known and new covering theorems can be used to construct a WB-covering;

b) how WB-coverings can be used to construct analogs of the Calderón-Zygmund decomposition for many spaces which are well-known in analysis (among them Triebel-Lizorkin spaces $F^s_{p,q}$ and Brudnyi's spaces of bounded variations);

c) some applications to real interpolation and singular integrals (will be discussed if time allows).