Rotundity properties in Calderón-Lozanovskiĭ spaces

Paweł Kolwicz

We will let (T, Σ, μ) be a $\sigma-$finite and complete measure space. By $L^0 = L^0(T)$ we denote the set of all μ-equivalence classes of real valued measurable functions defined on T. Given a *Köthe space* $(E, \|\cdot\|_E)$ and an *Orlicz function* φ we define on L^0 a convex modular I_φ by

$$I_\varphi(x) = \begin{cases} \|\varphi \circ x\|_E & \text{if } \varphi \circ x \in E, \\ \infty & \text{otherwise,} \end{cases}$$

where $(\varphi \circ x)(t) = \varphi(x(t))$, $t \in T$. By the *Calderón-Lozanovskiĭ space* E_φ we mean $E_\varphi = \{x \in L^0 : I_\varphi(cx) < \infty \text{ for some } c > 0\}$ equipped with so called *Luxemburg norm* defined by $\|x\|_\varphi = \inf(\lambda > 0 : I_\varphi(x/\lambda) \leq 1)$. It has been shown in [1, Remark 3] that geometry of Calderón-Lozanovskiĭ space E_φ can be "good" even if geometry neither of E nor of φ is "good". A couple of E and φ such that φ is not strictly convex and E is not strictly convex, but E_φ is uniformly convex, is presented in that paper. On the base of this phenomena the following natural problem arises.

Problem Find necessary and sufficient conditions for strict (uniform) convexity of Calderón-Lozanovskiĭ spaces.

Sufficient conditions for various properties of Calderón-Lozanovskiĭ spaces have been presented in several papers (see [3] for references). However, necessity of some among those conditions was only proved and it was concluded that some of sufficient conditions are not necessary. We shall find criteria for strict and uniform convexity of Calderón-Lozanovskiĭ spaces (see [3]) solving problem XII posed in [2]. In such a way it has been received an essential generalization of numerous criteria from papers concerning Orlicz-Lorentz spaces and improvements of theorems on Calderón-Lozanovskiĭ spaces.

References

[1] J. Cerdá, H. Hudzik and M. Mastyło, *On the geometry of some Calderón-Lozanovskiĭ interpolation spaces*, Indag. Math. N.S. 6(1), (1995), 35-49.

[2] S. Chen, Y. Cui, H. Hudzik and T. Wang, *On some solved and unsolved problems in geometry of certain classes of Banach function spaces*, Unsolved

Problems on Mathematics for the 21st Century, J. M. Abe&Tanaka (Eds.) IOS Press, 2001.

[3] P. Kolwicz, *Rotundity properties in Calderón-Lozanovskiĭ spaces,* Houston J. Math. 31.3 (2005), 883-912.

Institute of Mathematics, University of Technology, ul. Piotrowo 3a, 60-965 Poznań, Poland, e-mail : kolwicz@math.put.poznan.pl