Topological properties of two-modular convergence

Ewa Kasior

Let $(\emptyset, \Sigma, \mu)$ be a σ-finite measure space and assume that E is an ideal of L^0. Let ρ and ρ^* be two modulars defined on E, E_ρ and E_{ρ^*} the modular spaces for ρ and ρ^* respectively. In $E_\rho \cap E_{\rho^*}$ a two-modular convergence can be defined as follows: a sequence (f_n) in $E_\rho \cap E_{\rho^*}$ is said to be two-modular convergent to $f \in E_\rho \cap E_{\rho^*}$ whenever $f_n \to f$ with respect to modular ρ^* and (f_n) is ρ-bounded. We investigate a two-modular topology $\gamma_W(T_\rho, T_{\rho^*})$ in $E_\rho \cap E_{\rho^*}$ and we show that the convergence in this topology is equivalent to the two-modular convergence. We prove also that the two-modular convergence is equivalent to some modular convergence. Moreover, we obtained a characterization of linear functionals on the space $L^{\varphi_1} \cap L^{\varphi_2}$, continuous with respect to the two-modular topology $\gamma_W(T_{m_{\varphi_1}}, T_{m_{\varphi_2}})$. The functions φ_1 and φ_2 are not assumed to be convex.