ON SOME GEOMETRIC AND TOPOLOGICAL PROPERTIES OF GENERALIZED ORLICZ-LORENTZ SEQUENCE SPACES

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We will present results from [4] about Fatou property, order continuity, Kadec-Klee property with respect to the uniform convergence, some embeddings between a generalized Orlicz-Lorentz space λ_φ and their two subspaces, as well as its monotonicity and rotundity properties.

The triple $(N, 2^N, m)$ stands for the counting measure space, while $l^0 = l^0(m)$ denotes the space of all sequences $x : N \to (-\infty, \infty)$. For any $x \in l^0$ we define its distribution function $\mu_x : [0, +\infty) \to \{0, \infty\} \cup N$ by

$$\mu_x(\lambda) := m\{n \in N : |x_n| > \lambda\}$$

(see [2], [13] and [14]) and its nonincreasing rearrangement $x^* = (x_n^*)_{n=1}^\infty$ by

$$x_n^* := \inf\{\lambda : \mu_x(\lambda) < n\}.$$

Given any Musielak-Orlicz function $\varphi = (\varphi_n)_{n=1}^\infty$, we define on l^0 a convex modular ϱ_φ by

$$\varrho_\varphi(x) := \sup_\sigma \sum_{n=1}^\infty \varphi_n(x_{\sigma(n)}),$$

where σ denotes a permutation of the set N, the supremum is extended over all permutations of N and the modular space

$$\lambda_\varphi = \{x \in l^0 : \varrho_\varphi(\beta x) < \infty \quad \text{for} \quad \text{some} \quad \beta > 0\},$$

which becomes a normed space under the Luxemburg norm

$$\|x\| = \inf\{\beta > 0 : \varrho_\varphi\left(\tfrac{x}{\beta}\right) \leq 1\}.$$

Since the modular ϱ_φ is left continuous, we have that λ_φ has the Fatou property and consequently it is a rearrangement invariant Banach space.

The modular space λ_φ is called the generalized Orlicz-Lorentz space if $\varrho_\varphi(x) = \sum_{n=1}^\infty \varphi_n(x_n^*)$ for any $x \in \lambda_\varphi$ (cf [15]). Criteria in order that the modular space λ_φ is a generalized Orlicz-Lorentz space will be presented. The class of generalized Orlicz-Lorentz spaces is much more wide than the class of Orlicz-Lorentz spaces.

Results that will be presented are related to the results of Halperin [5] and Altshuler [1] concerning classical Lorentz spaces as well as to the results of Kamińska concerning Orlicz-Lorentz spaces (see [9] and [10]). We refer also to [6], [7], [3], [8], [11] and [12].

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