ON FIXED POINT THEOREMS IN STRICTLY CONVEX METRIC SPACES WITH ROUND BALLS

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We consider strictly convex metric space with round balls as special case of strictly convex metric space (see [1], [2], [3]).

Definition (I.Bula [1]). A strictly convex metric space (X, d) is said to *have round balls* if
$\forall a, b, c \in X \ (a \neq b) \ \forall z \in [a, b] \ (a \neq z \neq b)$ the following conditions holds

$$d(c, z) < \max\{d(c, a), d(c, b)\}. \tag{$*$}$$

It can be proved that condition

$$\forall a, b, c \in X \,(a \neq b)\, \forall z \in [a, b]\, (a \neq z \neq b) \quad d(c, z) \leq \max\{d(c, a), d(c, b)\}$$

is equivalent with condition of convexity of closed balls. This strict inequality (*) shows that if a and b belongs to sphere of ball $B(c, r)$, then z does not belong to this sphere, i.e., sphere does not contain straight lines therefore in Definition we speak about round balls.

In strictly convex metric space with round balls we can prove some fixed point theorems for nonexpansive mappings ([1]) and Kannan type mappings ([2]), for some commuting families of mappings too ([1], [2]). We prove some new fixed point theorems and consider sequences which converge to a fixed point.

References

[1] I.Bula, *Strictly convex metric spaces and fixed points*, Mathematica Moravica, **3**(1999), 5-16.

[2] I.Bula, *Strictly convex metric spaces with round balls and fixed points*, Orlicz Centenary Vol. II, Banach Center Publ., Vol. 68 (2005), P. 23-29.

[3] I.Bula, J.Vīksna, *Example of strictly convex metric spaces with not convex balls*, Int. J. of Pure and Applied Math., Vol. 25(1), 2005, P.87-93.