REPRESENTATION AND POSITIVITY OF JACOBI-ABEL KERNELS

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The Jacobi-Abel kernel for $\alpha > -1$, $\beta > -1$ we define as follows

$$K_r^{(\alpha,\beta)}(x,y) = \sum_{n=0}^{\infty} r^n \widehat{P}_n^{(\alpha,\beta)}(x)\widehat{P}_n^{(\alpha,\beta)}(y), \quad x, y \in [-1, 1],$$

where $0 < r < 1$ and $\widehat{P}_n^{(\alpha,\beta)}$, $n = 0, 1, \ldots$ are the Jacobi polynomials normalized in the respective L^2 space with the corresponding Jacobi weight. The main result is the representation for $K_r^{(\alpha,\beta)}$:

$$K_r^{(\alpha,\beta)}(x,y) = \sum_{m=0}^{\infty} c_m(r) Q_m^{(\alpha,\beta)}(x,y), \quad x, y \in [-1, 1],$$

where the coefficients $c_m(r) > 0$ are explicitly given and $\sum_{m=0}^{\infty} c_m(r) = 1$, and the $Q_m^{(\alpha,\beta)}$ are the Durrmeyer type positive kernels corresponding to the Jacobi orthogonal polynomials:

$$Q_m^{(\alpha,\beta)}(x,y) = \sum_{k=0}^{m} \lambda_{k,m}^{(\alpha,\beta)} \widehat{P}_k^{(\alpha,\beta)}(x)\widehat{P}_k^{(\alpha,\beta)}(y), \quad x, y \in [-1, 1],$$

where

$$\lambda_{k,m}^{(\alpha,\beta)} = \frac{\Gamma(m+1)\,\Gamma(m+\alpha+\beta+2)}{\Gamma(m-k+1)\,\Gamma(m+k+\alpha+\beta+2)}.$$