Conical Measures (Old and New)

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Conical measures were introduced by G. Choquet in the framework of his Classical Theorem of *Integral Representation.* If E is a locally convex Hausdorff space with continuous dual E', a *positive conical measure* μ on E is a positive linear form on the vector lattice of functions on E generated by E'. In this talk we review some recent applications of this theory and some older results.

If $X \subset E$ is a *weakly complete* convex cone (class $\mathcal{S}$) one of the main questions asked by G. Choquet is to find conditions on X ensuring that every μ carried by X is representable by a Radon measure on $X \setminus 0$ (subclass $\mathcal{L}$ of $\mathcal{S}$). We answered this question in several setting:

We proved that if $X \in \mathcal{L}$, then a positive conical measure on X can be defined as a positive form on the space of all continuous 1-homogeneous functions on X.

We proved that, if B is a Banach space, and if $X \in \mathcal{S}$ is contained in B', then ($X \in \mathcal{L}$ for the duality with B) iff (the norm on X is equivalent to an affine function).

Recently, we pointed out that, if B is a Banach space, to any linear operator $u : B' \to L^1$, one can associate canonically a conical measure μ_u on B; if $X \subset B$ is a convex cone then (μ_u is carried by X) iff (u is positive on $X^0 \subset B'$). This trick enables us to study the structure of X, namely to investigate the operators on B which are *summing* on X. Moreover, if X is *normal* we can find ℓ_n^p subspaces of B whose basis vectors are contained in X. In these results the index

$$i(X) = \inf\{p : (\sum ||x_i||^p)^{1/p} \leq C_p \sup || \sum \epsilon_i x_i||\}$$

where $x_i \in X$ and the sup is taken over all choice of signs has a capital role.

References: Cones convexes en analyse (Hermann, Paris): an English version will appear before long.