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**Recent developments on approximation by nonlinear integral operator  
in modular function spaces**

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More than hundred years ago I. Fredholm published in 1903 his famous paper on linear integral equations. Since then, linear integral operators have become important tools in many areas. In approximation theory applications were limited to linear operators because the notion of singularity of the kernel of a linear operator was closely connected with linearity. Starting about more than twenty years ago, Professor Julian Musielak introduced in [3] the first notion of singularity for a family of nonlinear integral operators of convolution type

$$(T_w f)(s) = \int_{\Omega} K_w(t, f(s+t)) d\mu(t),$$

defined on a suitable class of functions  $f : \Omega \rightarrow \mathbb{R}$ . Here  $\Omega$  is a compact interval of the real axis and  $(K_w)_{w>0}$  satisfies a uniform strong Lipschitz condition. Since then, the theory of approximation by nonlinear operators have been developed and extended in various directions, under the direction of J. Musielak and this lecture represents an abstract on the recent results obtained in collaboration with Julian Musielak and Gianluca Vinti during the years 1993-2003, (see [2]). In more recent years, the theory was furtherly generalized to abstract Urysohn type operators [1].

- [1] C. BARDARO and I. MANTELLINI, On global approximation properties of abstract integral operators in Orlicz spaces and applications, J. Ineq. Pure Appl. Math, **6**(4), Art.123, (2006).
- [2] C. BARDARO, J. MUSIELAK and G. VINTI, *Nonlinear integral operators and applications*, De Gruyter Series in Nonlinear Analysis and Appl., Vol.9, W. De Gruyter, Berlin, New York, 2003.
- [3] J. MUSIELAK, On some approximation problems in modular spaces, in *Constructive Function Theory 1981*, (Proc. Int. Conf. Varna, June 1-5, 1981), 455-461, Publ. House Bulgarian Acad. Sci, Sofia, 1983.