Isometric Decomposition Function Spaces and Applications to Nonlinear Evolution Equations[1]

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Abstract. Let $Q_0 = \{\xi \in \mathbb{R}^n : \xi_i \in [-1/2, 1/2),\ i = 1, ..., n\}$ and $Q_k = k + Q_0$, $k \in \mathbb{Z}^n$. It is easy to see that $\{Q_k\}_{k\in\mathbb{Z}^n}$ consists in a unit-cube decomposition of $\mathbb{R}^n$. Let $\mathscr{F}$ ($\mathscr{F}^{-1}$) be the (inverse) Fourier transform and χ_{Q_k} be the characteristic function on χ_{Q_k}. Denote $\Box_k \sim \mathcal{F}^{-1}\chi_{Q_k}\mathscr{F}$, $k \in \mathbb{Z}^n$, which are said to be the isometric decomposition operators. According to the isometric decomposition operators, we introduce a new class of function spaces $E^{\lambda}_{p,q}$ and $E_{s,p,q}$. For any $\lambda > 0, s \in \mathbb{R}$, $0 < p, q \leqslant \infty$, we write $\langle k \rangle = 1 + |k|$ and

$$\|f\|_{E_{s,p,q}} = \left(\sum_{k\in\mathbb{Z}^n} 2^{\lambda q|k|} \| \Box_k f\|^q_{L^p} \right)^{1/q};$$

$$\|f\|_{E_{s,p,q}} = \left(\sum_{k\in\mathbb{Z}^n} \langle k \rangle^{sq} \| \Box_k f\|^q_{L^p} \right)^{1/q}.$$

Applying these function spaces, we study the Cauchy problem for the nonlinear Schrödinger equation, the nonlinear Klein-Gordon equation and the Navier-Stokes equation. Some local and global well posedness results are obtained for the Cauchy data in the rough spaces $E_{0,2,1}$. In particular, we will obtain the Gevrey class regularity for the solutions of the Navier-Stokes equations.

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[1]This is a joint work with Professors B. Guo and H. Hudzik and L. Zhao.

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