Approximation problems in spaces of functions with bounded variation

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In [1] we obtain estimates, convergence results and rate of approximation for functions belonging to $BV-$spaces (spaces of functions with bounded variation) by means of nonlinear convolution integral operators. We treat both the periodic and the non-periodic case using, respectively, the classical Jordan variation and the multidimensional variation in the sense of Tonelli. In particular, in the non-periodic case, we consider the following family of nonlinear integral operators

$$(T_w f)(s) = \int_{\mathbb{R}} K_w(t, f(s-t))dt, \ w > 0, \quad s \in \mathbb{R}, \qquad (1)$$

for $f \in BV(\mathbb{R})$, denoting $BV(\mathbb{R})$ the space of functions with bounded variation, where $\{K_w\}_{w>0}$ is a family of measurable functions $K_w : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ of the form $K_w(t,u) = L_w(t)H_w(u)$ for every $t, u \in \mathbb{R}$, $\{H_w\}_{w>0}$ is a family of Lipschitz kernels $H_w : \mathbb{R} \to \mathbb{R}$ and $\{L_w\}_{w>0}$ is a family of approximate identities. For such operators, we mainly obtain a convergence result which proves that

$$V[T_w f - f] \to 0, \text{ as } w \to +\infty,$$

for $f \in BV(\mathbb{R})$ and we give a result on the degree of approximation for $V[T_w f - f]$ introducing suitable Lipschitz classes which take into account the functional of the space involved, i.e. the variation seminorm. Same results have been obtained in the periodic case and in the multidimensional setting of $BV(\mathbb{R}^N)$, $N \geq 1$.

In [2] we obtain similar results in the setting of functions with bounded $\varphi-$variation, being φ a $\varphi-$function. The concept of $\varphi-$variation in the sense of Musielak-Orlicz, which extends the classical variation, has been introduced by L.C. Young and it has been extensively studied by J. Musielak and W. Orlicz in [3]. Our main convergence result in this setting states that, under suitable assumptions on the family of the involved kernels, there holds $V_\varphi[\mu(T_w f - f)] \to 0$, as $w \to +\infty$, for some $\mu > 0$, being f a locally $\varphi-$absolutely continuous function and $\{T_w\}_{w>0}$ the family of integral operators (1).

References

[1] L. Angeloni and G. Vinti, *Convergence in Variation and Rate of Approximation for Nonlinear Integral Operators of Convolution Type,* to appear in Results in Math. (2006).

[2] L. Angeloni and G. Vinti, *Approximation by means of nonlinear integral operators in the space of functions with bounded $\varphi-$variation,* preprint (2006).

[3] J. Musielak and W. Orlicz, *On generalized variations (I),* Studia Mathematica, **18** (1959), 11–41.